MATERIAL DEFORMATION IN PRODUCTION

BMW Quantum Computing Challenge

Your way of bringing the foremost advances in quantum computing into real world challenges

BMW GROUP

Motivation

The modern automotive sector is highly competitive. Therefore, vehicle manufacturers aim to improve research and development and accelerate design and manufacturing processes by using innovative and efficient digital instruments and tools. One such instrument is digital modeling for automotive metal forming, which is an essential factor determining a vehicle's marketability, and also a necessary legal requirement. Automotive metal forming consists of compressing metal into a specific shape to create various vehicle's components, such as bumper beams, door frames, seat tracks, cross sills, to name a few (Figure 1). The process is based on the plastic deformation of metal sheets by a stamping press. Without such technology, car production would be an extremely challenging task[1].

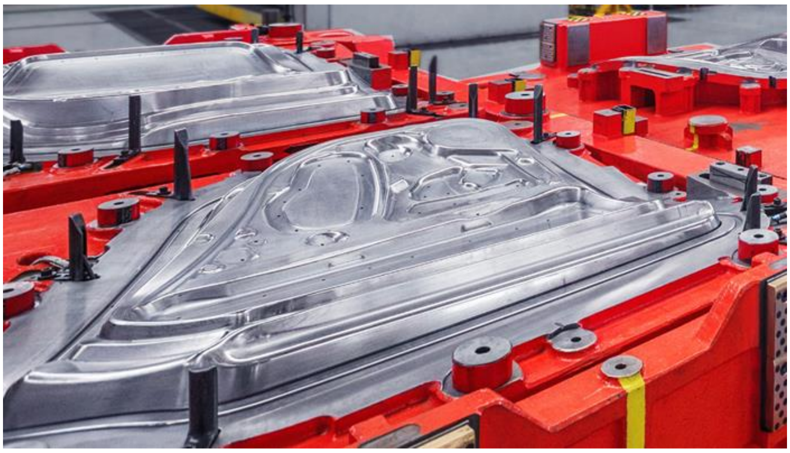

FIGURE 1 SHEET METAL FORMING DIE EXAMPLE (SOURCE: AUTOFORM).

For decades, physical testing for the deformation of prototype parts was the only reliable approach to verify the metal forming process requirements. Recently, virtual structural analysis and simulations became a promising alternative, quantitatively surpassing physical testing approaches in the vehicle production process. Robust and accurate simulations of material behaviour during the metal forming process can increase R&D productivity by enabling engineers to develop safer products. Simulations also reduce the time and effort required to evaluate alternative designs due to shorter feedback cycles.

Nowadays, numerical simulations of metal forming are actively used in the pre-production phase of vehicle components' manufacturing. The feasibility studies using finite element analysis are performed in very early design phases of sheet metal parts forming, predicting the geometries and properties of components, and optimizing them by varying the model parameters and the forming tools' shapes. Process simulation for these cases needs to consider numerous physical phenomena, including mechanical material properties and fracture behaviour at high forming speeds, and others. Using forming simulations allows for realistic representations of forming processes. It helps conduct feasibility studies and efficiently develop the optimal vehicle components that satisfy the cost and quality

[1] What is automotive metal forming (https://ltcroll.com/automotive-metal-forming).

requirements. Based on the numerical results, the manufacturing process and physical tool testing take place[2].

Reducing effort and expenditure for prototyping takes time and commitment. In this regard, fast, accurate and robust simulations play a central role, enabling automakers to reach new passive safety milestones throughout the product development process [1].

Classical solvers

Modern computing efficiency increased to a state where we can simulate physical deformation processes in realistic 3D models. Nevertheless, it is still essential to have a quality mathematical interpretation of the physical mechanism at this stage. A proper method will improve the modelling performance and accuracy and provide a better mastery of the mechanism itself characterization.

The widely-used classical numerical methods for structurral analysis are the grid-based techniques, which interpolate the initial continuous system of Partial Differential Equations (PDE) on a grid of 3D points. Finite Difference Method (FDM), Finite Volume Method (FVM), Finite Element Method (FEM), Spectral Method (SM), and Discontinuous FEM are techniques used to derive the spatial derivative operators' discrete representation. The main difference between them lies in the way they represent the exact solution by an approximate one and how this approximate solution satisfies the PDE. In FDM, this representation is trivial and intuitive, requiring fewer degrees of freedom and uniform meshes, making the approach less expensive in the class. However, more sophisticated and costly discretization techniques, such as DG FEM, provide better accuracy and convergence for models with complex geometries and high heterogeneities. Some generic properties of the methods mentioned above regarding the different numerical criteria are shown in Table 1 [2].

Over the years, FEM became the most popular automotive metal forming simulation tool for

Numerical method	Complex geometries	High-order accuracy and hp-adaptivity	Explicit semi-discrete form	Conservation laws	Elliptic problems
FDM	-	+	+	+	+
FVM	+	-	+	+	(+)
FEM	+	+	-	(+)	+
DG FEM	+	+	+	+	(+)

TABLE 1 GENERIC PROPERTIES OF THE MOST WIDELY-USED NUMERICAL METHODS ('+' REPRESENTS SUCCESS, '-' - SHORT-COMING IN THE METHOD, '(+)' - METHOD CAN BE USED BUT IT REMAINS LESS NATURAL CHOICE).

its balance between numerical cost and accuracy. In BMW, some of the forming simulations

[2] Simulation in forming technology (https://www.iwu.fraunhofer.de/content/dam/iwu/en/documents/Brochures/IWU-KB-Simulation-in-Forming-Technology.pdf).

(similar to the crash modelling) are performed using LS-DYNA commercial solver, based on the explicit FEM model[3]. Realistic 3D simulations of different vehicle components (especially those with complex geometries) are highly challenging and numerically costly (require several days/weeks), still providing with numerical drawbacks due to the imperfect approximation. To minimize the problem size, engineers are often obliged to employ several simplifications such as 2D-shell elements for the discretization, rigid forming tools and simplified material models instead of realistic 3D ones. It reduces the simulation time to hours; however, the simplified model's predictability is also highly impacted. A robust numerical approach will allow increasing the efficiency and predictability of numerical simulations while aiming at zero-prototyping for sustainable vehicle components' manufacturing. Using Quantum Computing to enhance the convenient classical approaches may potentially accelerate computations, improving the accuracy and enabling larger and more realistic model.

Quantum computing approach

One of the earliest and well-known attempts to solve the linear systems of equations faster and more efficient by the means of Quantum Computing was made by Harrow, Hassidim, and Lloyd [3]. However, regarding the Noisy Intermediate-Scale Quantum (NISQ) devices' limitations, the proposed algorithm (HHL) is still a long-term goal, mainly because it requires a Quantum Random Access Memory (QRAM) or another efficient tool for generating an analog-encoded quantum state.

In the near-term perspective, there have been some advancements in the development of variational algorithms for solving the linear systems of equations - Variational Quantum Linear Solver (VQLS) [4], and for finding the spectrum of eigenvalues and eigenvectors for a density matrix - Variational Quantum State Diagonalisation (VQSD) [5]. Both algorithms demonstrate good robustness to the NISQ restrictions, although they lack an efficient state preparation routine to decompose the initial classical matrices into quantum states' products. It is an essential step to preserve the algorithm's quantum advantage with respect to its classical counterparts.

Most promising recent developments of quantum algorithms for solving non-linear PDEs was done by the group of scientists from the University of Oxford and University of Singapore in 2019 [6] and the researchers from the University of Exeter and Qu\&Co in 2020 [7].

The first approach is based on the tensor networks programming model and and it uses the multiple copies of variational quantum states to treat the non-linearities efficiently. In the second work authors introduce Differentiable Quantum Circuits (DQC) resulting from an analytical derivation of the functions defined as expectation values of the parametrized quantum circuit, thus avoiding the discretization error accumulation. The DQCs are trained to satisfy PDEs and specific boundary conditions.

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[3] How BMW uses Ansys LS-DYNA to predict vehicle behaviour in collisions (https://develop3d.com/simulation/ansys-lsdyna-bmw-predict-vehicle-behaviour-in-collisions).

Governing equations

From the mathematical point of view the problem consists in solving of system of PDEs, based on the continuity equation, the equation of motion, the dissipation inequality, along with appropriate conditions and an adequate constitutive relation for the Cauchy stress tensor.

To briefly describe the governing equations let us consider a solid body in its onloaded condition R and the body forces b acting on this body per unit volume. Let us consider the boundary conditions, specifying displacements $u^*(x)$ on a portion $\partial_1 R$ or tractions on a portion $\partial_2 R$ of the boundary ∂R of R. The problem consists in calculating the displacements u_i, strains ε_{ij} and stresses σ_{ij} which satisfy the following equations [8]:

- Displacement equation:

$$\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right);$$

- The equation of static equilibrium for stresses:

$$\frac{\partial \sigma_{ij}}{\partial x_i} + b_j = 0;$$

- The boundary conditions on displacement and stress:

$$u_i = u_i^* \ on \ \partial_1 R, \quad \sigma_{ij} n_i = t_j^* \ on \ \partial_2 R;$$

- The hypoelastic constitutive law, which relates stress to strain as follows:

$$\sigma_{ij} = S_{ij} + \frac{\sigma_{kk}\delta_{ij}}{3}, \quad S_{ij} = \frac{2}{3}\sigma_e \frac{e_{ij}}{\varepsilon_e}, \quad \sigma_{kk} = \frac{E}{1-2\nu}\frac{1}{3}\varepsilon_{kk},$$

where

$$e_{ij} = \varepsilon_{ij} - \frac{1}{3}\varepsilon_{kk}\delta_i, \quad \varepsilon_e = \sqrt{\frac{2}{3}e_{ij}e_{ij}},$$

$$\frac{\sigma_e}{\sigma_0} = \begin{cases} \sqrt{\frac{1+n^2}{(n-1)^2} - \left(\frac{n}{n-1} - \frac{\varepsilon_e}{\varepsilon_0}\text{\big}\right)^2} - \frac{1}{n-1}, & \text{if } \varepsilon_e \le \varepsilon_0 \\ \left(\frac{\varepsilon_e}{\varepsilon_0}\right)^{\frac{1}{n}}, & \text{otherwise} \end{cases}$$

and $E = n\sigma_0/\varepsilon_0$ is the slope of the uniaxial stress-strain curve at $\varepsilon_e = 0$.

The descriptions for material constants n, σ_0 and ε_0 for hypoelastic constitutive law can be found in [8] (section 3.5). Note that this material model does not describe any actual material, but is sometimes used to approximate the more complicated stress-strain laws for plastic materials. Governing equations in terms of the Virtual Work Principle as well as an example of FEM discretization are proposed in [8] (section 8.3).

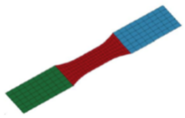
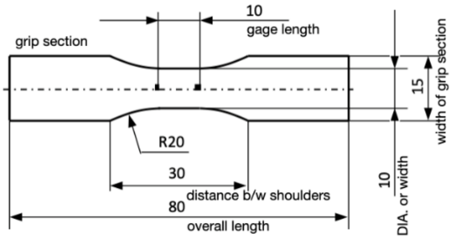
Test type	Standard	Specimen geometry and dimension
Tensile test	DIN EN ISO 8256:2005 Type 3 	

FIGURE 2 SPECIMEN MODEL USED FOR NUMERICAL SIMULATIONS (SOURCE: BMW, EP-40).

Using more sophisticated models and approximations is not prohibited for developing a future numerical approach and its validation on a test example proposed below. The principal metrics would consist of comparison (together with the specialists of BMW) of the computation results for concrete discretization models with state-of-the-art industrial solutions and the level of innovation built on top of the existing established classical and quantum approaches.

Test model

A tensile test, or so-called tension test, is one of the most fundamental and common mechanical testing types to determine materials' mechanical properties. By applying the pulling (tensile) force to a material (specimen) and measuring the specimen's response to the stress, engineers can define the material's maximal elongation and robustness.

A tensile specimen generally represents a standardized sample cross-section. It has two shoulders and a gage in between. The shoulders must be large enough to be gripped, whereas the gage section has a small cross-section to ensure the deformation and failure occur in this area (Figure 2) [9].

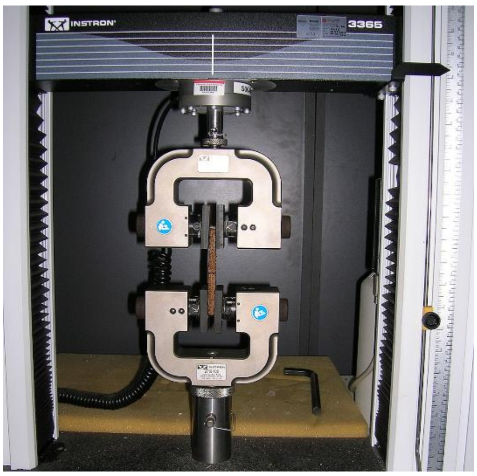


FIGURE 3 EXPERIMENTAL REALISATION OF UNI-AXIAL TENSILE TEST.

During the tensile test, the specimen is placed in the testing machine and slowly extended until fracturing (Figure 3).

The gage section's elongation is recorded against the applied force during the test. The data is manipulated so that it is not specific to the geometry of the test sample. The elongation measurement (the change from initial to final gage length $\Delta L = L - L_0$) is used to calculate the engineering strain $\varepsilon = \Delta L / L_0$. The tensile force measurement F_n and the specimen's nominal cross-section A are used to calculate the engineering stress $\sigma = \frac{F_n}{A}$. The machine performs these calculations as the force increases, enabling building the corresponding stress-strain curve and showing the material's reaction on the applied force. The point of break or failure is of engineers' interest, including other important properties such as modulus of elasticity, yield strength, and strain. An extensive description of the tensile test process, equipment, results is proposed in [9].

The proposed test geometry, as well as a choice of material, are not mandatory. It can be used as a validation example to build a discretization accordingly to the employed numerical technique and the simulation capacities of hardware (open choice).

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